

Exercise 55

Find the derivative of the function. Simplify where possible.

$$h(t) = \cot^{-1}(t) + \cot^{-1}(1/t)$$

Solution

Use the chain rule and the derivatives of the inverse trigonometric functions listed on page 214.

$$\begin{aligned}\frac{dh}{dt} &= \frac{d}{dt} \left[\cot^{-1}(t) + \cot^{-1} \left(\frac{1}{t} \right) \right] \\ &= \frac{d}{dt} \cot^{-1}(t) + \frac{d}{dt} \cot^{-1} \left(\frac{1}{t} \right) \\ &= \left(-\frac{1}{1+t^2} \right) + \left[-\frac{1}{1+\left(\frac{1}{t}\right)^2} \cdot \frac{d}{dt} \left(\frac{1}{t} \right) \right] \\ &= -\frac{1}{1+t^2} + \left[-\frac{1}{1+\frac{1}{t^2}} \cdot \left(-\frac{1}{t^2} \right) \right] \\ &= -\frac{1}{1+t^2} + \left[\frac{1}{\left(1+\frac{1}{t^2}\right)t^2} \right] \\ &= -\frac{1}{1+t^2} + \left(\frac{1}{t^2+1} \right) \\ &= 0\end{aligned}$$